Generalized Mathematical Modeling and Analysis of a PMDC Generator

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Abstract

This paper presents the generalized mathematical modeling of permanent magnet dc generator using state space approach. Transfer function of the generator is derived with the help of derived state model. The performance of PMDC generator under various conditions is simulated using MATLAB/SIMULINK environment and simulation result demonstrates the feasibility of the proposed system. A proportional controller is used to get better response from the designed system in MATLAB environment.

Key words: state space, PMDC generator, mathematical modeling, MATLAB/SIMULINK

1. Introduction

A dc generator is an electrical machine which converts mechanical energy into direct current electricity. This energy conversion is based on the principle of production of dynamically induced emf. According to Faraday's laws of electromagnetic induction, whenever a conductor is placed in a varying magnetic field (OR a conductor is moved in a magnetic field), an emf (electromotive force) gets induced in the conductor. The magnitude of induced emf can be calculated from the emf equation of dc generator. If the conductor is provided with the closed path, the induced current will circulate within the path. In a dc generator, field coils produce an electromagnetic field and the armature conductors are rotated into the field. Thus, an electromagnetically induced emf is generated in the armature conductors. The direction of induced current is given by Fleming's right hand rule.

The Permanent Magnet DC Generator can be considered as a separately excited DC brushed generator with a constant magnetic flux. The PMDC generator consists of stator having rare earth permanent magnets such as Neodymium or Samarium cobalt to produce a very strong stator field flux instead of field coils and a commutator connected through brushes to a wound armature. This type of generator is generally light in weight, more reliable, higher efficiency and can operate at low operational speeds. There are no field windings in stator, therefore the field coil losses are zero.

2. PMDC Generator Modeling using State Space Analysis

The different equations related to DC generator are given below

$$E_g = k_g w_r \varphi \tag{1}$$
$$V_t = E_g - L_a \frac{dI_a}{dt} - R_a I_a \tag{2}$$
$$T_L = k_L \varphi I_a \tag{3}$$
$$\mathrm{T_{shaft}} = \mathrm{T_L} + J\frac{\mathrm{dw_r}}{\mathrm{dt}} + \mathrm{Bw_r} \tag{4}$$
$$V_t = I_a R_L \tag{5}$$

Where V_t = terminal voltage, E_g= total generated voltage, L_a = armature inductance, R_a = armature resistance, I_a = armature current, k_g = voltage constant, φ = Stator/field flux, $\mathrm{w_r}$ = speed of the generator, R_L = load resistance, $\mathrm{T_{shaft}}$ = shaft torque, T_L = load torque, J = moment of inertia, B = viscous friction, k_L = load torque.
The necessary differential equations will now be derived by using above equations to simulate the dc generator.

$$\frac{dI_a}{dt} = \frac{k_g \varphi w_r}{L_a} - \frac{(R_a + R_L)}{L_a} I_a \tag{6}$$
$$\frac{dw_r}{dt} = \frac{k_L \varphi I_a}{J} - \frac{B}{J} w_r + \frac{T_{shaft}}{J} \tag{7}$$

Here the generator terminal voltage is controlled by varying the generator shaft torque. Hence $\mathrm{T_{shaft}}$ is the input variable and V_t is the output variable.
We chose as the state variables $x_1(t) = I_a$ and $x_2(t) = w_r$ (8)
The state equations will now be derived by using above equations.

$$\frac{dx_1(t)}{dt} = -\frac{(R_a + R_L)}{L_a} x_1(t) + \frac{k_g \varphi}{L_a} x_2(t) \tag{9}$$
$$\frac{dx_2(t)}{dt} = -\frac{k_L \varphi}{J} x_1(t) - \frac{B}{J} x_2(t) + \frac{T_{shaft}}{J} \tag{10}$$
$$y(t) = V_t = I_a R_L \tag{11}$$

Hence state model of dc motor is derived from equations (9), (10) and (11) as follows

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{(R_a + R_L)}{L_a} & \frac{k_g \varphi}{L_a} \\ -\frac{k_L \varphi}{J} & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} u(t) \qquad (12)$$

$$y(t) = [R_L \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \qquad (13)$$

3. PMDC Generator State Model Simulation Using MATLAB

The set of model equations given by ((9), (10) and (11)) are solved to compute the instantaneous values of the performance variables of the system. Let, the PMDC generator parameters (coefficient of differential equations) are assigned to be J = 0.01kg-m2, B = 0.1N-m sec/rad, R_a= 1 Ω, L_a= 0.4 H, k_g= 0.27 V-sec/rad, k_L= 0.11 N-m/A, R_L= 3Ω.
Simulation has been carried out by varying different shaft torques (20 N-m, 30N-m) and keeping field flux constant (0.30 T) of the PMDC generator. The simulation results are shown in following figures.

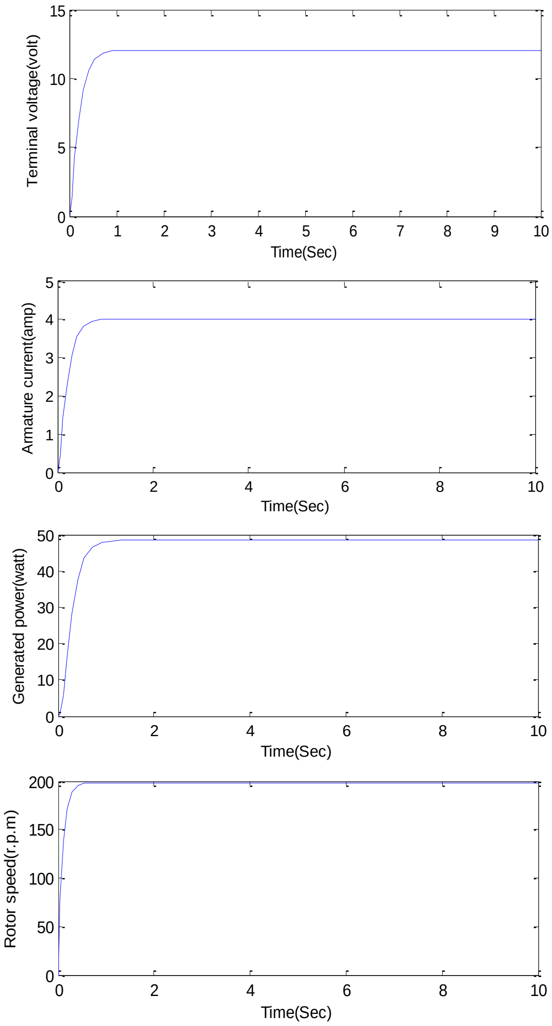

Fig: 1 Response of PMDC generator with Tshaft = 20 N-m and field flux (φ) = 0.30 T (constant)

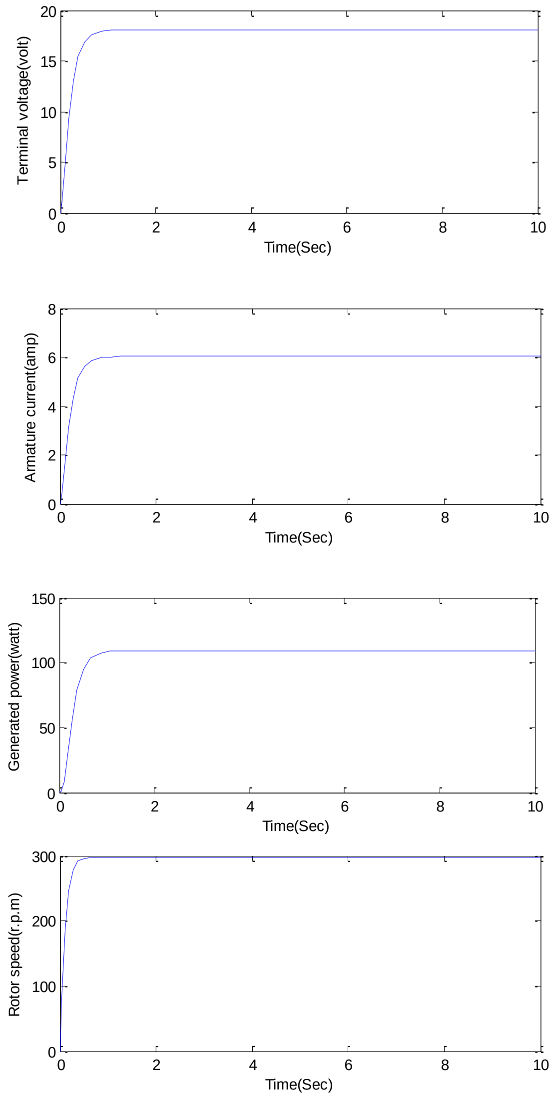

Fig: 2 Response of PMDC generator with Tshaft = 30 N-m and field flux (φ) = 0.30 T (constant)

4. PMDC Generator State Model Using Generator Parameters

The state model of PMDC generator is derived using generator parameters and equation (12) and (13) as follows:

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} -10 & 0.20 \\ -3.3 & -10 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 100 \end{bmatrix} u(t) \qquad (14)$$

$$y(t) = [3 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \qquad (15)$$

Using equation (14) and (15), we get
$A = \begin{bmatrix} -10 & 0.20 \\ -3.3 & -10 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 100 \end{bmatrix}$,

C =[3   0] , D = [0]   (16)

5. PMDC Generator Modeling Using Transfer Function Approach

The transfer function of the generator is derived from the state model by taking initial condition as zero as follows:

T(S) = $C(SI - A)^{-1}B + D$ (17)

The following MATLAB program is used to get the transfer function of the generator

```
>> A = [-10 0.20;-3.3 -10];
B = [0; 100]; C = [3 0]; D = [0];
[num, den] = ss2tf (A, B, C, D)

num =

     0     0    60

den =

    1.0000   20.0000   100.6600
```

So, the transfer function of the generator is given by

T(S) = $\frac{60}{S^2+20S+100.66}$ (18)

6. Step Response of the Derived Model

The following MATLAB program is used to get the step response of the derived state model

```
>> A = [-10 0.20;-3.3 -10];
B = [0; 100]; C = [3 0]; D = [0];
Step (A, B, C, D)
```

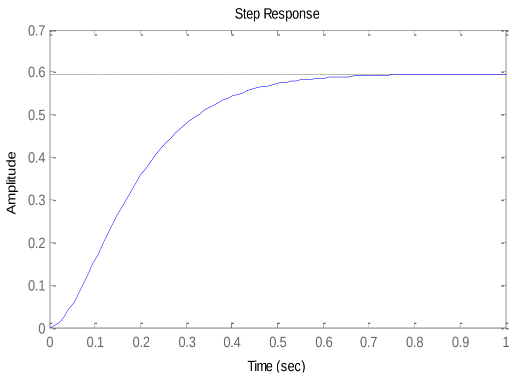

Fig: 3 Step response of the PMDC generator

7. Modified Response of the Model

A proportional controller (Kp = 1.7) is used to get better response for the system.

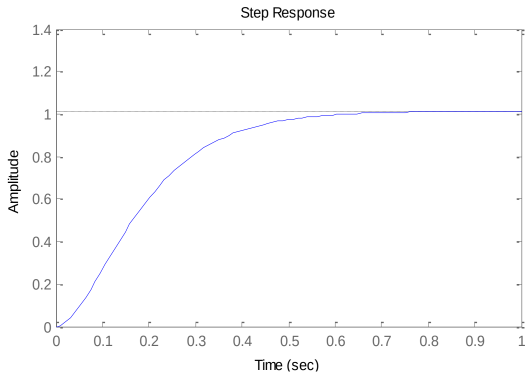

Fig: 4 Modified Step response of the PMDC generator

8. Conclusion

The state model of PMDC Generator has been developed and simulation has been carried out for the proposed system. The step response of the derived system is also checked. This paper will help undergraduate students to explore more about PMDC generator.

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